Generation of macroscopic singlet states in atomic ensembles

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DPG March Meeting, Hannover
9 March, 2010

Outline

1. Motivation
2. Spin squeezing and entanglement
3. Spin squeezing with atomic ensembles
4. Von Neumann measurement

Outline

Motivation

- In many quantum experiments the qubits cannot be individually addressed. We still would like to create and detect entanglement.

- Entanglement creation and detection is possible through spin squeezing. We will use the ideas behind the spin squeezing approach in order to
  - Create and detect entanglement between particles with arbitrarily large spin
  - Engineer quantum states other than the classical spin squeezed state with a large spin, that is, unpolarized states.
  - Generalize the Gaussian approach for describing the dynamics leading to such states.

Outline

Entanglement

Definition

Fully separable states are states that can be written in the form

$$\rho = \sum_l p_l \rho_l^{(1)} \otimes \rho_l^{(2)} \otimes ... \otimes \rho_l^{(N)},$$

where $\sum_l p_l = 1$ and $p_l > 0$.

Definition

A state is entangled if it is not separable.

The standard spin-squeezing criterion

Definition

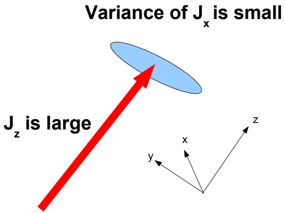
The spin squeezing criterion for entanglement detection is

$$\frac{(\Delta J_x)^2}{\langle J_y\rangle^2 + \langle J_z\rangle^2} \geq \frac{1}{N}.$$

If it is violated then the state is entangled.

[A. Sørensen, L.M. Duan, J.I. Cirac, P. Zoller, Nature **409**, 63 (2001).]

- Note that this criterion is for spin-$1/2$ particles.
- States violating it are like this:



A generalized spin squeezing entanglement criterion

Separable states of N spin-j particles must fulfill

$$\xi_s^2 := (\Delta J_x)^2 + (\Delta J_y)^2 + (\Delta J_z)^2 \geq Nj.$$

ξ_s is zero for many-body singlet states.

[GT, PRA **69**, 052327 (2004);GT, C. Knapp, O. Gühne, and H.J. Briegel, PRL **99**, 250405 (2007).]

- $N\xi_s^2$ gives an upper bound on the number of unentangled spins.
- ξ_s^2 characterizes the sensitivity to external fields acting as $U = \exp(i\phi J_{\mathbf{n}})$.
- $\xi_s = 0$ corresponds to complete insensitivity.

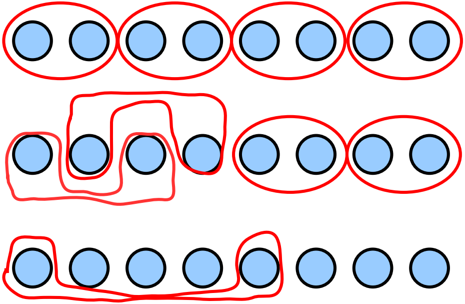
Many-body singlet states

Many-body singlet states: important in condensed matter physics and quantum information science.

- Metrological applications for gradient measurements.
- Quantum memory for the decoherence free subspace.
- Here we realize singlets without two-spin interactions or waiting for a Heisenberg system to settle in ground state of a Heisenberg system.

Permutationally invariant singlet

- Our singlet is the equal mixture of all permutations of a pure singlet state.
- For qubits, it is the mixture of all chains of two-qubit singlets:



- Such a state has intriguing properties ...

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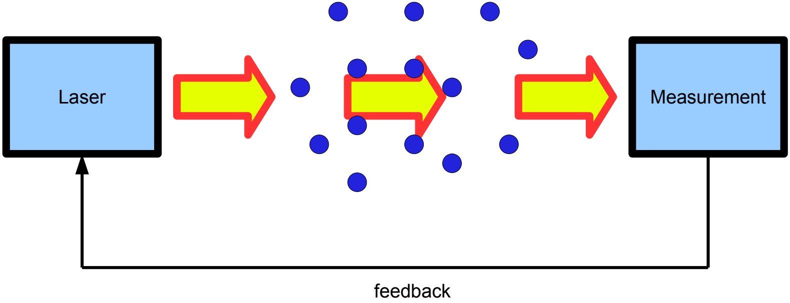
The physical system: atoms + light

- We consider atoms interacting with light. [B. Julsgaard, A. Kozhekin, and E.S. Polzik, Nature **413**, 400 (2001); S.R. de Echaniz, M.W. Mitchell, M. Kubasik, M. Koschorreck, H. Crepaz, J. Eschner, and E.S. Polzik, J. Opt. B **7**, S548 (2005); J. Appel, P.J. Windpassinger, D. Oblak, U.B. Hoff, N. Kjaergaard, and E.S. Polzik, arXiv:0810.3545.]

- The light is then measured and the atoms are projected into an entangled state.



Quantum non-demolition measurement (QND) of the ensemble

The steps the the QND measurement of J_k :

- 1. Set the light to
$$\langle \mathbf{S} \rangle = (S_0, 0, 0).$$
- 2. The atoms interact with the light for time t
$$H = \Omega J_k S_z$$
- 3. Measurement of S_y.

- The most obvious effect of such a measurement is the decrease of $(\Delta J_k)^2$.
- The timescale of the dynamics, for $J := Nj$, is
$$t \sim \tau := \frac{1}{\Omega \sqrt{S_0 J}}.$$

The proposed protocol

1. Initial state
    - Atoms

$$\varrho_0 := \frac{\mathbb{1}}{(2j+1)^N}$$

    - Light

$$\langle \mathbf{S} \rangle = (S_0, 0, 0).$$

2. Measurement of J_x + feedback or postselection.
3. Measurement of J_y + feedback or postselection.
4. Measurement of J_z + feedback or postselection.

- We consider 10^6 spin-1 ^{87}Rb atoms and $S_0 = 0.5 \times 10^8$.
- Initial state of the atoms has $(\Delta J_k)^2 \sim N$ for $k = x, y, z$.
- After squeezing, we obtain $\xi_s < 1$.
- Thus, we get a state close to a singlet state.

Covariance matrix

- We define the set of operators

$$R = \{\frac{J_x}{\sqrt{J}}, \frac{J_y}{\sqrt{J}}, \frac{J_z}{\sqrt{J}}, \frac{S_x}{\sqrt{S_0}}, \frac{S_y}{\sqrt{S_0}}, \frac{S_z}{\sqrt{S_0}}\}$$

and covariance matrix as

$$\Gamma_{mn} := \langle R_m R_n + R_n R_m \rangle / 2 - \langle R_m \rangle \langle R_n \rangle.$$

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- For short times, the dynamics of an operator O_0 is given by

$$O_P = O_0 - it[O_0, H],$$

  where we assumed $\hbar = 1$.

Covariance matrix II

- Consider dynamics for $t \sim \tau := \frac{1}{\Omega\sqrt{JS_0}}$.
- For these times, for the unitary dynamics one arrives to

$$\Gamma_P = M\Gamma_0 M^T,$$

where M is the identity matrix, apart from $M_{5,1} = \frac{\langle S_x \rangle}{S_0}\kappa$, and $\kappa := t/\tau = \Omega t\sqrt{JS_0}$.

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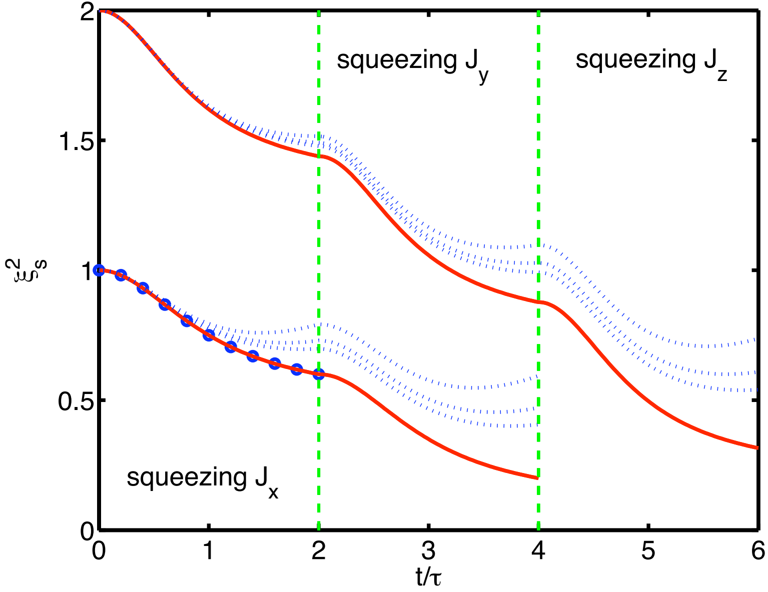
- The measurement of the light can be modeled with a projection

$$\Gamma_M = \Gamma_P - \Gamma_P (P_y \Gamma_P P_y)^{\mathrm{MP}} \Gamma_P^T,$$

where MP denotes the Moore-Penrose pseudoinverse, and P_y is $(0, 0, 0, 0, 1, 0)$. [G. Giedke and J.I. Cirac, Phys. Rev. A **66**, 032316 (2002).]

Spin squeezing dynamics (top curve, solid)



Modeling losses

The dynamics of the covariance matric for the case of losses

$$\Gamma'_P = (\mathbb{1} - \eta D) M \Gamma_0 M^T (\mathbb{1} - \eta D) + \eta(2-\eta) D \Gamma_{\mathrm{noise}},$$

where $D = \mathrm{diag}(1,1,1,0,0,0)$ and $\Gamma_{\mathrm{noise}} = \mathrm{diag}(\frac{2}{3},\frac{2}{3},\frac{2}{3},0,0,0)$.

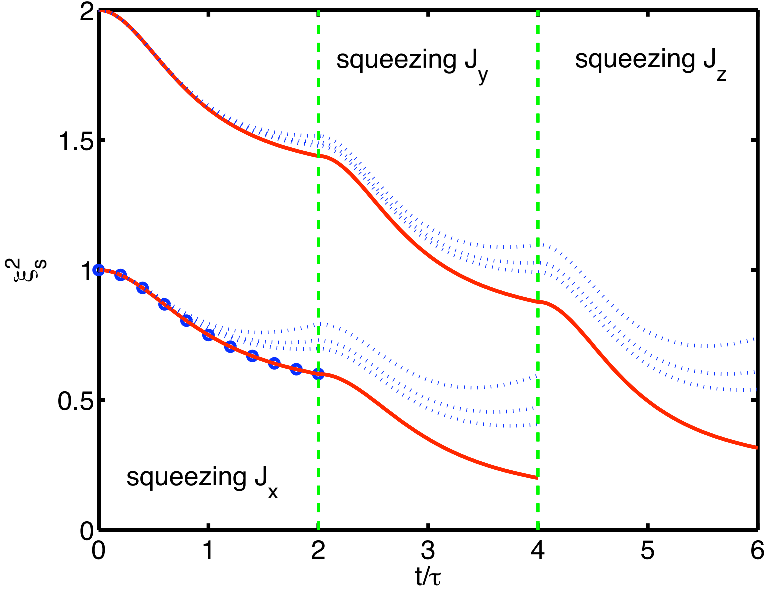
- η the fraction of atoms that decoherence during the QND process.
- The losses are connected to κ through

$$\eta = Q\kappa^2/\alpha,$$

where α is the resonant optical depth of the sample and $Q = \frac{8}{9}$

[L.B. Madsen and K. Mølmer, Phys. Rev. A **70**, 052324 (2004).]

Spin squeezing dynamics: $\alpha = 50, 75, 100$ (dotted)



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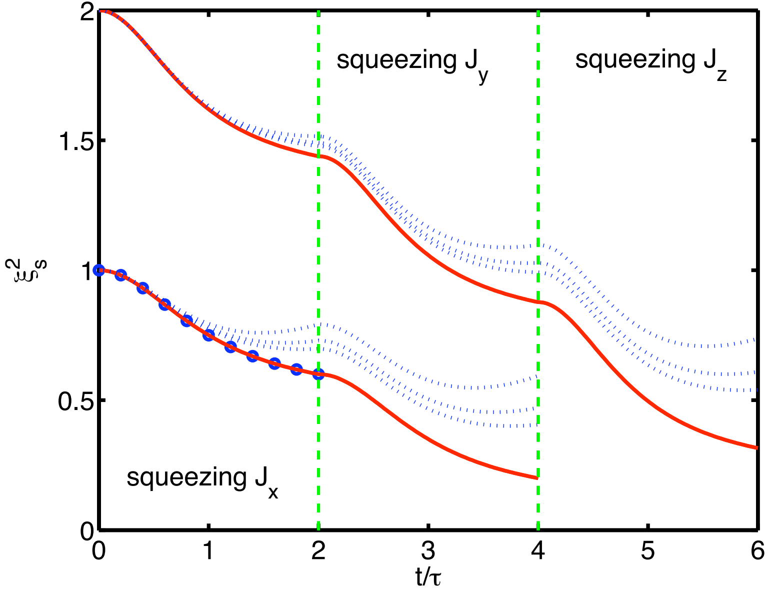
Exact model

Results: for $t \sim \tau \times N^{\frac{1}{4}}$ the variances decrease to $\sim \sqrt{N}$, while for $t \sim \tau \times \sqrt{N}$ the variances reach ~ 1, which we call the von Neumann limit.

- Direct simulation of a system with a million atoms is not possible.
- However, in the large N limit, a formalism can be obtained that replaces sums by integrals.
- Works also for the regime in which the Gaussian approximation is no more valid.
- Comparison for exact model is possible for an initial state for which half of the spins are in the $|+1\rangle_x$ state, half of them are in the $|-1\rangle_x$ state.

Spin squeezing dynamics (bottom curve, dots)



Conclusions

- We presented a method for creating and detecting entanglement in an ensemble of atoms with spin $j > \frac{1}{2}$.
- Our experimental proposal aims to create a many-body singlet state through squeezing the uncertainties of the collective angular momenta.
- We showed how to use an extension of the usual Gaussian formalism for modeling the experiment.
- Presentation based on: GT and M.W. Mitchell, arxiv:0901.4110.

*** THANK YOU ***