Witnessing metrologically useful multiparticle entanglement

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Why multipartite entanglement and metrology are important?

- Full tomography is not possible, we still have to say something meaningful.
- Claiming “entanglement” is not sufficient for many particles.
- We should tell
  - How entangled the state is
  - What the state is good for, etc.

Outline

Entanglement

A state is (fully) separable if it can be written as

$$\sum_k p_k \varrho_k^{(1)} \otimes \varrho_k^{(2)} \otimes ... \otimes \varrho_k^{(N)}.$$

If a state is not separable then it is entangled (Werner, 1989).

k-producibility/k-entanglement

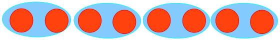
A pure state is k-producible if it can be written as

$$|\Phi\rangle = |\Phi_1\rangle \otimes |\Phi_2\rangle \otimes |\Phi_3\rangle \otimes |\Phi_4\rangle ....$$

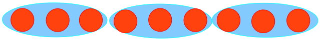
where $|\Phi_l\rangle$ are states of at most k qubits.

A mixed state is k-producible, if it is a mixture of k-producible pure states.
[e.g., Gühne, GT, NJP 2005.]

- If a state is not k-producible, then it is at least $(k+1)$-particle entangled.



two-producible



three-producible

Outline

Many-particle systems for j=1/2

- For spin-$\frac{1}{2}$ particles, we can measure the collective angular momentum operators:

$$J_l := \frac{1}{2} \sum_{k=1}^{N} \sigma_l^{(k)},$$

where $l = x, y, z$ and $\sigma_l^{(k)}$ are Pauli spin matrices.

- We can also measure the variances

$$(\Delta J_l)^2 := \langle J_l^2 \rangle - \langle J_l \rangle^2.$$

Outline

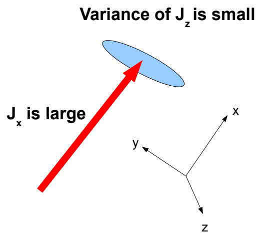
The standard spin-squeezing criterion

Spin squeezing criteria for entanglement detection

$$\xi_s^2 = N \frac{(\Delta J_z)^2}{\langle J_x \rangle^2 + \langle J_y \rangle^2}.$$

If $\xi_s^2 < 1$ then the state is entangled.
[Sørensen, Duan, Cirac, Zoller, Nature (2001).]

- States detected are like this:



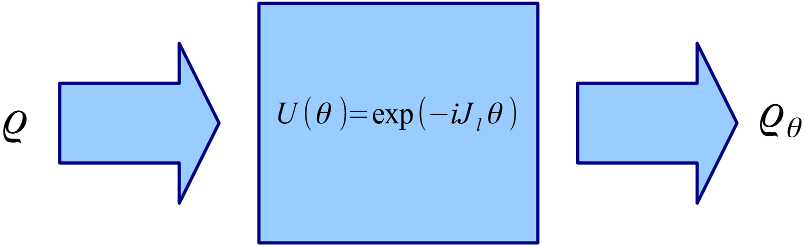
- See talks about recent spectacular experiments at this conference.

Outline

1. Introduction and motivation
2. Spin squeezing and entanglement
   - Entanglement
   - Collective measurements
   - The original spin-squeezing criterion
3. Detecting metrologically useful entanglement
   - Basics of quantum metrology
   - Witnessing metrological usefulness
   - Metrology with measuring $\langle J_z \rangle$
   - Metrology with measuring $\langle J_z^2 \rangle$
   - Metrology with measuring any operator

Quantum metrology

- Fundamental task in metrology with a linear interferometer



- We have to estimate θ in the dynamics

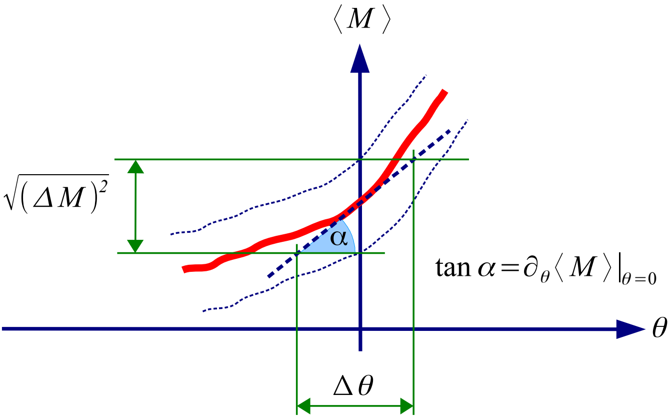
$$U = \exp(-iJ_l\theta)$$

where $l \in \{x, y, z\}$.

Precision of parameter estimation

- Measure an operator M to get the estimate θ. The precision is

$$(\Delta\theta)^2 = \frac{(\Delta M)^2}{|\partial_\theta \langle M \rangle|^2}.$$



The quantum Fisher information

- Cramér-Rao bound on the precision of parameter estimation

$$(\Delta\theta)^2 \geq \frac{1}{F_Q[\varrho, A]}, \qquad (\Delta\theta)^{-2} \leq F_Q[\varrho, A].$$

where $F_Q[\varrho, A]$ is the quantum Fisher information.

- The quantum Fisher information is given by an explicit formula for ϱ and A.

The quantum Fisher information vs. entanglement

- For separable states

$$F_Q[\varrho, J_l] \le N.$$

[Pezze, Smerzi, PRL 2009; Hyllus, Gühne, Smerzi, PRA 2010]

- For states with at most k-particle entanglement (k is divisor of N)

$$F_Q[\varrho, J_l] \le kN.$$

[Hyllus *et al.*, PRA 2012; GT, PRA 2012].

- If a state violates the above inequality then it has $(k+1)$-particle metrologically useful entanglement.

Metrological precision vs. entanglement

- For separable states

$$(\Delta\theta)^{-2} \le N.$$

[Pezze, Smerzi, PRL 2009; Hyllus, Gühne, Smerzi, PRA 2010]

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   - Metrology with measuring any operator

Witnessing metrological usefulness

- Direct measurement of the sensitivity
  - Measure $(\Delta\theta)^2$.
  - Obtain bound on F_Q and multipartite entanglement.
  - Experimentally challenging, since we need dynamics.
  - The precision is affected by the noise during the dynamics.

- Witnessing (our choice)
  - Estimate how good the precision were, if we did the metrological process.
  - Assume a perfect metrological process. Characterizes the state only.

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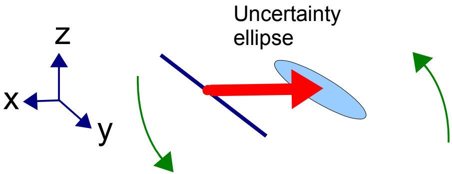
Metrology with spin-squeezed states

- Pezze-Smerzi bound

$$(\Delta\theta)^2 = \frac{(\Delta J_z)^2}{|\partial_\theta \langle J_z\rangle|^2} = \frac{(\Delta J_z)^2}{\langle J_x\rangle^2} = \frac{\xi_s^2}{N}.$$

- We measure $\langle J_z\rangle$.



[Pezze, Smerzi, PRL 2009.]

Outline

Metrology with Dicke states

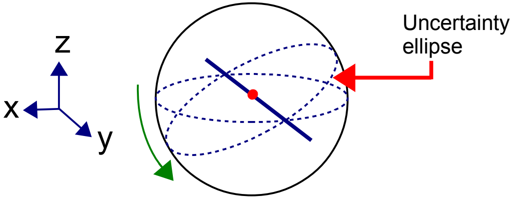
- For Dicke state

$$\langle J_l \rangle = 0, l = x, y, z, \quad \langle J_z^2 \rangle = 0, \quad \langle J_x^2 \rangle = \langle J_y^2 \rangle = \text{large}.$$

- Linear metrology

$$U = \exp(-iJ_y\theta).$$

- Measure $\langle J_z^2 \rangle$ to estimate θ. (We cannot measure first moments, since they are zero.)



Formula for maximal precision II

Maximal precision with a closed formula

$$(\Delta\theta)^2_{\rm opt} = \frac{2\sqrt{(\Delta J_z^2)^2(\Delta J_x^2)^2} + 4\langle J_x^2\rangle - 3\langle J_y^2\rangle - 2\langle J_z^2\rangle(1+\langle J_x^2\rangle) + 6\langle J_z J_x^2 J_z\rangle}{4(\langle J_x^2\rangle - \langle J_z^2\rangle)^2}.$$

- Collective observables, like in the spin-squeezing criterion.
- Metrological usefulness can be verified without carrying out the metrological task.
- Tested on experimental data.

[Apellaniz, Lücke, Peise, Klempt, GT, NJP 2015.]

Outline

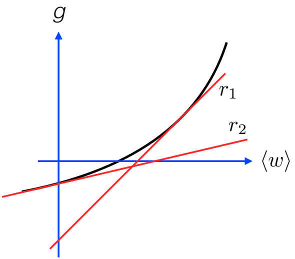
Large step: we do not assume any metrological scheme

- We would like to know how good a state is for quantum metrology.
- We allow any operator to be measured for parameter estimation.
- Thus, we need to witness the quantum Fisher information.

Legendre transform

- Optimal linear lower bound on a convex function $g(\varrho)$ based on an operator expectation value $w = \langle W \rangle_\varrho = \mathrm{Tr}(W\varrho)$

$$g(\varrho) \geq rw - const.,$$



where $w = \mathrm{Tr}(\varrho W)$.

- For every slope r there is a "*const.*"

- Textbooks say

$$g(\varrho) \geq \mathcal{B}(w) := rw - \hat{g}(rW),$$

where $\hat{g}$ is the Legendre transform

$$\hat{g}(W) = \sup_\varrho [\langle W \rangle_\varrho - g(\varrho)].$$

[Gühne, Reimpell, Werner, PRL 2007; Eisert, Brandao, Audenaert, NJP 2007.]

Legendre transform II

- Bound is best if we optimize over r as

$$g(\varrho) \geq \mathcal{B}(w) := \sup_r \left[rw - \hat{g}\left(rW \right) \right],$$

where again $w = \mathrm{Tr}(\varrho W)$.

- F_Q is the convex roof of the variance. Hence, it is sufficient to carry out an optimization over pure states

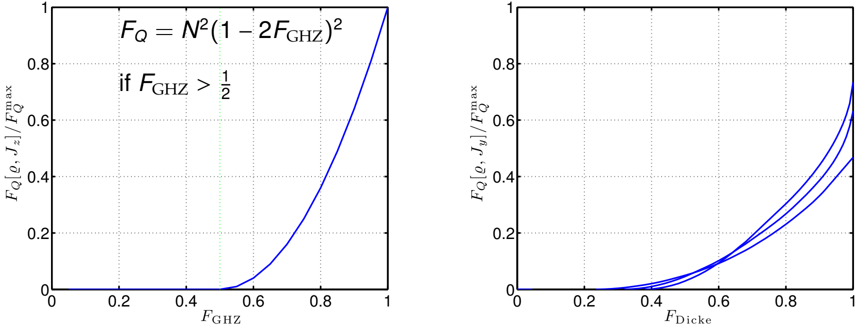
$$\hat{g}(W) = \sup_\Psi [\langle W \rangle_\Psi - g(\Psi)].$$

[GT, Petz, PRA 2013; S. Yu, arXiv1302.5311 (2013);
GT, Apellaniz, J. Phys. A: Math. Theor. 2014]

- With further simplifications, an optimization over a single (!) real variable is needed.

Witnessing the quantum Fisher information based on the fidelity

- Let us bound the quantum Fisher information based on some measurements. First, consider small systems. [See also Augusiak *et al.*, 1506.08837.]



Quantum Fisher information vs. Fidelity with respect to (a) GHZ states and (b) Dicke states for $N = 4, 6, 12$.

[Apellaniz *et al.*, arXiv:1511.05203.]

Bounding the qFi based on collective measurements

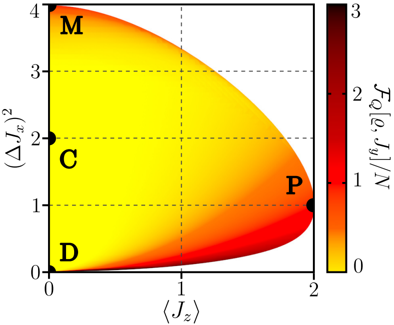
Bound for the quantum Fisher information for spin squeezed states (Pezze-Smerzi bound)

$$F_Q[\varrho, J_y] \geq \frac{\langle J_z \rangle^2}{(\Delta J_x)^2}.$$

[Pezze, Smerzi, PRL 2009.]

Bounding the qFi based on collective measurements II

- Optimal bound for the quantum Fisher information $F_Q[\varrho, J_y]$ for spin squeezing for $N = 4$ particles

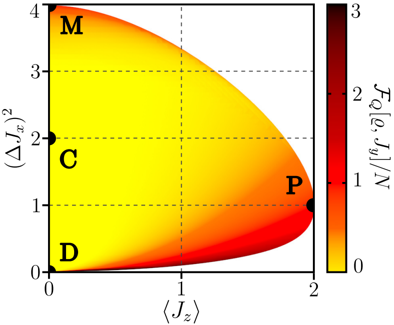


P=fully polarized state, D=Dicke state, C=completely mixed state, M=mixture of $|00..000\rangle_x$ and $|11..111\rangle_x$

[Apellaniz, Kleinman, Gühne, GT, arXiv:1511.05203.]

Bounding the qFi based on collective measurements III

- Optimal bound for the quantum Fisher information $F_Q[\varrho, J_y]$ for spin squeezing for $N = 4$ particles

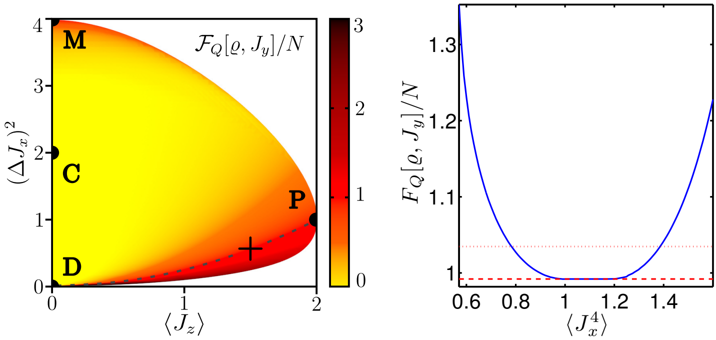


On the bottom part of the figure ($(\Delta J_x)^2 < 1$) the bound is very close to the Pezze-Smerzi bound!

[Apellaniz, Kleinman, Gühne, GT, arXiv:1511.05203.]

Bounding the qFi based on collective measurements IV

- The bound can be obtained if additional expectation value, i.e., $\langle J_x^2 \rangle$ is measured, or we assume symmetry:



[Apellaniz, Kleinman, Gühne, GT, arXiv:1511.05203.]

Spin squeezing experiment

- Experiment with $N = 2300$ atoms,

$$\xi_s^2 = -8.2\mathrm{dB} = 10^{-8.2/10} = 0.1514.$$

[Gross, Zibold, Nicklas, Esteve, Oberthaler, Nature 2010.]

- The Pezze-Smerzi bound is:

$$\frac{\mathcal{F}_Q[\varrho_N, J_y]}{N} \geq \frac{1}{\xi_s^2} = 6.605.$$

- We get the same value for our method!

[Pezze, Smerzi, PRL 2009]

- Similar calculations for Dicke state experiments!

[Lücke, Peise, Vitagliano, Arlt, Santos, Tóth, Klempt, PRL 2014.]

Summary

- We discussed a very flexible method to detect multipartite entanglement and metrological usefulness.
- We can choose a set of operators and the method gives an optimal lower bound on F_Q.

Apellaniz, Lücke, Peise, Klempt, GT, New J. Phys. 17, 083027 (2015);
Apellaniz, Kleinmann, Gühne, Tóth, arxiv: arXiv:1511.05203.

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