Quantum Information Class 2024

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See https://www.gtoth.eu/courses.html

- Introduction
- General characteristics of multi-partite quantum systems
  - Statistics of classical bits, probabilistic logic by von Neumann
  - Quantum bits - a comparison with the statistics of classical bits
  - Geometry of quantum quantum states
    - Bloch vector for $d = 2$ (a single qubits)
    - Bloch vector for $d > 2$
  - Convexity of the set of quantum states
    - Quantum states with full rank are inside the set, with derivation
    - Quantum states with non-full rank are on the boundary, with derivation
  - Reduced states, partial trace
  - Schmidt decomposition for bipartite pure states
  - Purification
  - Entropy
    - Shannon entropy
    - Von Neumann entropy, main properties.
      - Prove concavity with Klein's inequality
      - prove that it is additive for independent systems
      - Prove subadditivity.
      - Araki-Lieb inequality (mention)
      - Strongly subadditive (mention)
    - Quantum conditional entropy
    - Quantum mutual information

    - Quantum relative entropy, properties, including monotonicity under CP maps.
    - Linear entropy, relation to von Neumann entropy via the Mercator series
  - Quantum Fidelity and its properties
  - Distances (Bures distance, Hilbert-Schmidt distance, Trace distance)

- Interesting quantum states
  - Single-particle states
  - Bipartite singlet state, invariance under $U \otimes U$
  - Werner state, invariance under $U \otimes U$, full set
  - Isotropic state. invariance under $U \otimes U^*$
  - Schrödinger cat state
  - GHZ state and its stabilizer group (xxx,zz1,z1z,1zz,etc.)
  - W state
  - Symmetric Dicke state and its defining properties

- Bell inequalities
  - EPR paradox
  - LHV models
  - CHSH inequality
  - Mermin's inequality
  - Loopholes (Detection efficiency loophole, locality loophole)

- Paper review: Gregor Weihs, Thomas Jennewein, Christoph Simon, Harald Weinfurter, and Anton Zeilinger, Violation of Bell's Inequality under Strict Einstein Locality Conditions, Phys. Rev. Lett. 81, 5039 (1998)

- Paper review: J. Yin, Y. Cao, Y.-H. Li et al., Satellite-based entanglement distribution over 1200 kilometers, Science 356, 1140-1144 (2017)

- Entanglement theory (entangled/not entangled)
  - Bipartite case
    - Pure states
    - Mixed states (Werner 1989)
  - Entanglement criteria
    - Partial transposition, with derivation

        * Entanglement witnesses, several examples, e.g., witness derived for any pure state
        * Variance-based criteria, e.g., the well known criterion $\mathrm{var}(x_1 - x_2) + \mathrm{var}(p_1 + p_2) > 2$.
        * Multipartite case

- Entanglement measures (How much is it entangled?)
    - General quantum operation
    - Local operations and classical communication (LOCC)
    - Entanglement of formation
    - Concurrence
    - Entanglement of distillation
    - Bound entanglement (prove that it cannot be distilled to singlets, and its overlap with singlets is small)
    - Requirements for entanglement measures
    - Negativity

- No-cloning theorem and related issues
    - No cloning theorem
    - Measurement problem
    - Quantum teleportation
    - Quantum money
    - Quantum cryptography (BB84, E91)
    - Error correction (bit flip code, phase-flip code, Shore code)

- Paper review: D. Bouwmeester, J.-W. Pan, K. Mattle, M. Eibl, H. Weinfurter, A. Zeilinger, Experimental quantum teleportation, Nature 390, 575-579 (1997).

- Quantum metrology
    - Simple examples of quantum metrology
        * Classical case: Clock arm
        * Quantum case: Single spin-1/2 particle
        * Magnetometry with the fully polarized state
        * Magnetometry with the spin-squeezed state
        * Metrology with the GHZ state
        * Dicke states

        - Interferometry with squeezed photonic states
    - Entanglement theory
        - Multipartite entanglement
        - The spin-squeezing criterion
    - Quantum metrology using the quantum Fisher information
        - Quantum Fisher information
        - Quantum Fisher information in linear interferometers
        - Noise and imperfections

- Paper review: Event-Ready Bell Test Using Entangled Atoms Simultaneously Closing Detection and Locality Loopholes, Wenjamin Rosenfeld, Daniel Burchardt, Robert Garthoff, Kai Redeker, Norbert Ortegel, Markus Rau, and Harald Weinfurter Phys. Rev. Lett. 119, 010402 (2017).

- Semidefinite programming
    - Linear programming
    - Semidefinite programming
    - Solvable vs. not solvable by SDP
    - Basic optimization tasks solvable by SDP, for instance,
        - N-representability problem (on small systems)
        - Maximum of an operator expectation value for a set given by constraints
        - Maximum of an operator expectation value for PPT states
        - Minimizing an expression given as the sum of expectation value squares

Literature:

- I. Bengtsson and K. Zyczkowski, Geometry of Quantum States, An Introduction to Quantum Entanglement, Cambridge University Press, 2009.

- R. Horodecki, P. Horodecki, M. Horodecki, and K. Horodecki, Quantum entanglement, Rev. Mod. Phys. 81, 865 (2009).

- O. Gühne and G. Tóth, Entanglement detection, Phys. Rep. 474, 1 (2009); arxiv:0811.2803.

- G. Tóth and I. Apellaniz, Quantum metrology from a quantum information science perspective, J. Phys. A: Math. Theor. 47, 424006 (2014), special issue "50 years of Bell's theorem"; arxiv:1405.4878.