

**Violation of Bell's Inequality under Strict
Einstein Locality Conditions
— we review a paper
(Lecture of the Quantum Information class of
the Master in Quantum Science and
Technology)**

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Why is photonic implementation of quantum information processing interesting?

- Advantages

- Photons are inherently two-state systems, they can have a horizontal or vertical polarization.
- They can easily be transmitted in a fibre, even for 100 kilometers.
- There are simple processes that generate entangled photon pairs.
- Highly entangled states can be realized with large fidelity.

- Drawbacks

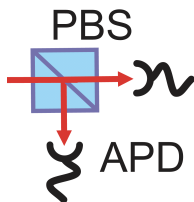
- Photons do not interact with each other, thus it is not so simple to realize quantum gates.
- The efficiency of photonic detectors is not large, however, it getting better recently.

Photons as qubits

- A photon can have a horizontal and a vertical polarization.
- $|H\rangle$ and $|V\rangle$ can take the role of $|0\rangle$ and $|1\rangle$.
- The photon can travel even 100 kilometers on the fiber.

Detecting the qubit

- We have to use Polarizing Beam Splitters (PBS). These let photons with a H polarization go through, while they reflect photons with a V polarization.
- Based on these, the following setup measures the photon in the H/V basis (essentially the same as measuring the Pauli spin matrix σ_z)



- APD means single-photon avalanche photodiodes. These are detectors that can detect even a single photon.

Figure from [W. Wieczorek, R. Krischek, N. Kiesel, P. Michelberger, G. Tóth, and H. Weinfurter, Phys. Rev. Lett. 2009.]

Detecting the qubit II

- How to measure in some other basis? How to measure σ_x or σ_y ?
- We have to add half-wave plates and quarter wave plates before the detector.
- This way, we get a detector that can measure σ_x , σ_y , and σ_z .

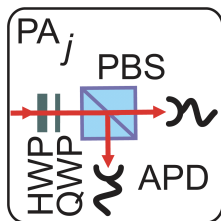
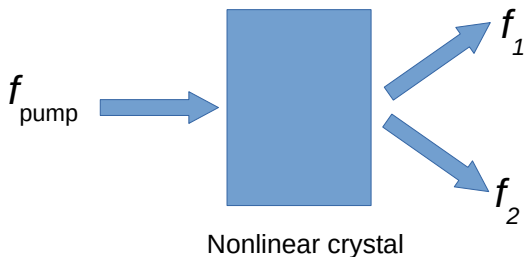


Figure from [W. Wieczorek, R. Krischek, N. Kiesel, P. Michelberger, G. Tóth, and H. Weinfurter, Phys. Rev. Lett. 2009.]

Parametric downconversion

- It is possible to create a pair of entangled photons from a single photon.
- This is called spontaneous parametric downconversion (SPDC).
- The crystal is a nonlinear crystal, e.g., a BBO (beta-barium borate) crystal.



Parametric downconversion II

- The process respects energy conservation

$$f_{\text{pump}} = f_1 + f_2$$

and momentum conservation

$$k_{\text{pump}} = k_1 + k_2$$

for the incoming photon and the outgoing photons. Here f are frequencies and k are wave vectors.

- Typically,

$$f_1 = f_2 = f_{\text{pump}}/2.$$

Hence, the frequency of the incoming photon is twice the frequency of the two outgoing photons.

- The process is probabilistic, its probability can be, e.g., 10^{-6} .
- A laser beam, called "pump" beam is directed into the crystal. Most of the photons pass through the crystal. However, sometimes the photon is down-converted to a photon pair.

Parametric downconversion III

- Depending on, where the outgoing photons exit, there are several types of SPDC.
- For Type II SPDC, the photons have opposite polarizations. If this is also a colinear Type II SPDC, then they pass via the same fibre (rather than one of them going this way, the other going that way.)
- Such a process gives a state

$$\frac{1}{\sqrt{2}}(|HV\rangle + |VH\rangle).$$

This is a symmetric state. It is maximally entangled, similarly to $\frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ and $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$.

- Thus, we can create a maximally entangled state.

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Violation of Bell's Inequality under Strict Einstein Locality Conditions

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(Received 6 August 1998)

We observe strong violation of Bell's inequality in an Einstein-Podolsky-Rosen-type experiment with independent observers. Our experiment definitely implements the ideas behind the well-known work by Aspect *et al.* We for the first time fully enforce the condition of locality, a central assumption in the derivation of Bell's theorem. The necessary spacelike separation of the observations is achieved by sufficient physical distance between the measurement stations, by ultrafast and random setting of the analyzers, and by completely independent data registration. [S0031-9007(98)07901-0]

PACS numbers: 03.65.Bz

The stronger-than-classical correlations between entangled quantum systems, as first discovered by Einstein, Podolsky, and Rosen (EPR) in 1935 [1], have ever since occupied a central position in the discussions

the directions of polarization analysis were switched after the photons left the source. Aspect *et al.*, however, used periodic sinusoidal switching, which is predictable into the future. Thus communication slower than the speed

Photonic experiment without a locality loophole I

- G. Weihs, T. Jennewein, C. Simon, H. Weinfurter, and A. Zeilinger, *Violation of Bell's Inequality under Strict Einstein Locality Conditions*, [Phys. Rev. Lett. 81, 5039 \(1998\)](#).

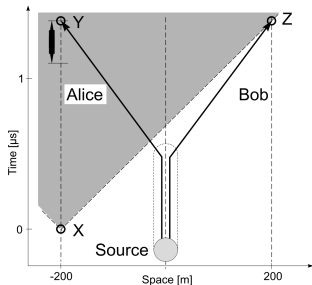


FIG. 1. Spacetime diagram of our Bell experiment. Selecting a random analyzer direction, setting the analyzer, and finally detecting a photon constitute the measurement process. This process on Alice's side must fully lie inside the shaded region which is invisible to Bob's during his own measurement. For our setup this means that the decision about the setting has to be made after point "X" if the corresponding photons are detected at spacetime points "Y" and "Z", respectively. In our experiment the measurement process (indicated by a short black bar) including the choice of a random number took less than only one-tenth of the maximum allowed time. The vertical parts of the kinked photon world lines emerging from the source represent the fiber coils at the source location, which are obviously irrelevant to the locality argument.

Photonic experiment without a locality loophole III

- In the photonic experiments, the horizontal/vertical (H/V) polarization of the photon encodes the logical 0 and 1, respectively.
- The state analyzed is

entangled state $|\Psi\rangle = 1/\sqrt{2}(|H\rangle_1|V\rangle_2 + e^{i\varphi}|V\rangle_1|H\rangle_2)$,
which we chose $\varphi = \pi$.

That is, we have

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle).$$

- The qubit can be measured with polarizing beam splitters (PBS) and detectors. On the following figure, PBS is called "Polarizer".

Photonic experiment without a locality loophole IV

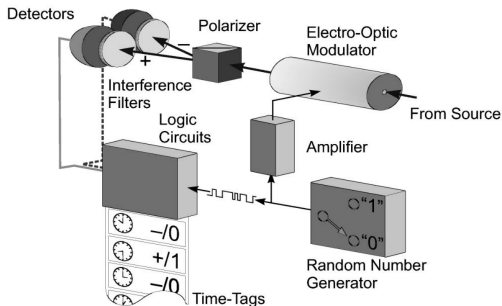


FIG. 2. One of the two observer stations. A random number generator is driving the electro-optic modulator. Silicon avalanche photodiodes are used as detectors. A “time tag” is stored for each detected photon together with the corresponding random number “0” or “1” and the code for the detector “+” or “-” corresponding to the two outputs of the polarizer.

Photonic experiment without a locality loophole V

- The experiment tests the CHSH inequality:

coincidence rates. In a rather general form the CHSH inequality reads

$$S(\alpha, \alpha', \beta, \beta') = |E(\alpha, \beta) - E(\alpha', \beta)| \\ + |E(\alpha, \beta') + E(\alpha', \beta')| \leq 2. \quad (1)$$

- The correlations are defined as follows:

$E(\alpha, \beta)$ of the correlation between Alice's and Bob's local results is $E(\alpha, \beta) = [C_{++}(\alpha, \beta) + C_{--}(\alpha, \beta) - C_{+-}(\alpha, \beta) - C_{-+}(\alpha, \beta)]/N$, where N is the sum of all coincidence rates. In a rather general form the CHSH

This correlation term is like $\langle \sigma_z \otimes \sigma_z \rangle$ or $\langle \sigma_{\vec{n}_1} \otimes \sigma_{\vec{n}_2} \rangle$, where \vec{n}_k are some spin directions.

Photonic experiment without a locality loophole VI

Summary of the experiment:

- CHSH experiment with two photons, without the locality loophole.
- Note that the detection loophole has not been solved that time.